EARTHQUAKE RESPONSE OF ASYMMETRIC BUILDINGS:

A PARAMETRIC STUDY

by

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ABSTRACT

Results of a parametric study on earthquake time history response of asymmetric single storey structures with one axis of symmetry modelled as two degree-of-freedom systems are presented. A large number of such systems with a range of mass center to rigidity center eccentricities, a range of uncoupled lateral natural frequencies and a number of torsional to lateral frequency ratios Ω_o were subjected to several earthquake acceleration records. Five percent damping was assumed in the two coupled modes. Maximum displacements at several locations along the roof deck were computed, normalized with respect to the symmetric case, averaged, and compared with code oriented static methods. The study shows that the static approach does not give reliable estimates for the response of frames in asymmetric buildings, even when the amplification factors provided by earthquake codes are incorporated into the formulation. In particular, code provisions usually underestimate the response of frames located on the side of the rigidity center away from the mass center for small to moderate eccentricities when $\Omega_o^2 = 0.5, 1.0$; whereas for systems with higher torsional rigidities ($\Omega_o^2 = 2.0$), the static approach appears to yield reasonable results. For members located on the opposite side of the roof, code provisions, as well as a recently proposed modification thereto, appear to underestimate the response with increasing frequency ratio.

INTRODUCTION

The torsional motion of asymmetric building structures during earthquakes can be attributed to two major sources: (1) torsional ground motion; (2) coupling of lateral and torsional oscillations resulting from either designed or accidental eccentricity in the structure or the mass distribution. Although several earthquake codes have incorporated some recommendations on torsional coupling since the late 1950's, these effects are not yet well understood. This situation is reflected in the large number of publications on torsional response of structures published in the technical literature during the last twenty years (1). This paper considers coupling effects due to lateral excitation only in structures having one axis of symmetry. Since the response of generally asymmetric structures in the direction of excitation is usually larger when x-y coupling is neglected (2), this simplified model is conservative.

The torsional provisions of earthquake codes are based on the traditional static method of seismic analysis in which the inertia forces are applied statically to the structure at the mass center. For asymmetric structures this approach is problematic, since modal lateral-torsional coupling due to the rotatory inertia of the deck magnifies the effective eccentricity of the inertia forces, and affects members located on opposite sides of the rigidity center to a different extent. This coupling effect is usually accounted for in the seismic codes (e.g. ref. 3) by specifying a factor α multiplying the static eccentricity e, and by assigning two values to this factor: $\alpha>1.0$ for members located on the mass center CM side of the rigidity center CR (flexible side), and $\alpha<1.0$ for members on the opposite or stiff side (Fig. 1). Other effects, which are not dealt with in this paper, namely, torsional input, accidental eccentricity and other imponderables, are considered by means of an additional eccentricity which usually is given as a fraction β of the width of the building b.

Thus, the design or dynamic eccentricity e_d in most codes is given in the form:

$$e_d = \alpha e \pm \beta b \qquad (1)$$

in which αe = the dynamic eccentricity,and βb = the additional or accidental eccentricity.

However, with all these refinements, the fact remains that the computed response of any given frame or assemblage in the system, even when the computation is based on the theoretically correct dynamic eccentricity, rather than the code values, does not correlate with those obtained from spectrum analysis when applying the RSS (square root of the sum of square) formula to the frame's modal responses, or with the results of time history analyses. In short, the factored static methods in their present form either overestimate, or underestimate the response in an inconsistent manner. Thus, the amplification factor for eccentricity is in fact a variable rather than a constant (or two), its numerical value depending on the location of the frame in question (4,5).

Therefore, if static methods are to continue in use, formulae more consistent with dynamic analysis should be adopted by earthquake codes. The purpose of the present paper is to compare the earthquake response of structural members in simple single storey asymmetric structures dynamically analyzed for several time histories, with results based on code oriented static analyses. These comparisons will enable to evaluate the adequacy of accepted static and quasi-dynamic procedures, as well as of a recently proposed modification to the static code approach (5), and help in formulating expressions which better correlate with reported dynamic results.

TWO DEGREES OF FREEDOM SYSTEMS

The system studied is an idealized single storey structure shown in Fig. 1, consisting of a rigid deck with mass m supported laterally by several massless planar assemblages (e.g. flexural walls or frames). For simplicity, one axis of symmetry is assumed, so that only two DoF (degrees of freedom) are considered, namely, the lateral displacement y, and the rotation θ about a vertical axis through the mass center. The lateral and torsional rigidities K_y and K_θ of the 2-DoF system are obtained from the stiffnesses K_{iy} of the individual members, or assemblages (if they are not simple columns or walls) in the usual way, namely:

$$K_y = \sum_i K_{iy} \qquad (2)$$

$$K_\theta = \sum_i K_{iy}\, x_i^2 + \sum_i K_{ix}\, y_i^2 + \sum K_{i\theta} = K_{\theta 0} + K_y e^2$$

in which x_i and y_i are the perpendicular distances to the mass center, $K_{i\theta}$ = the torsional rigidity of a member about its own axis (may be neglected for planar members or assemblages) and $K_{\theta 0}$ = torsional rigidity of the system about the center of rigidity. The eccentricity e of the center of rigidity CR from the mass center CM is given by:

$$e = \frac{1}{K_y} \sum_i K_{iy}\, x_i \qquad (3)$$

The earthquake acceleration time history $\ddot{U}_g(t)$ is assumed to act in the y direction only. The equations of motion of the system in the linear range with respect to the mass center are given by (e.g. ref. 4):

$$\begin{Bmatrix} \ddot{y} \\ \rho\ddot{\theta} \end{Bmatrix} + \omega_y^2 \begin{bmatrix} 1 & e^* \\ e^* & \Omega_M^2 \end{bmatrix} \begin{Bmatrix} y \\ \rho\theta \end{Bmatrix} = - \begin{Bmatrix} \ddot{U}_g \\ 0 \end{Bmatrix} \qquad (4)$$

in which m = storey (or deck) mass, ρ = mass radius of gyration about CM,

$$\omega_y = \sqrt{K_y/m} \;;\; \omega_{\theta 0} = \sqrt{K_{\theta 0}/m\rho^2} \;;\; \omega_\theta = \sqrt{K_\theta/m\rho^2}$$

$$e^* = e/\rho \;;\; \Omega_0^2 = \omega_{\theta 0}^2/\omega_y^2 \;;\; \Omega_M^2 = \omega_\theta^2/\omega_y^2 = \Omega_0^2 + e^{*2}$$

Note that, in order to render equation 4 dimensionally compatible, $\rho\theta$ rather than θ has been taken as the rotational variable.

For modal analysis, it is necessary to assume proportional damping, therefore, the damping terms do not appear in the equations of motion. They are evaluated for the uncoupled system in the usual way, namely:

$$C = \bar{\alpha}M + \bar{\beta}K \qquad (5)$$

where C,M,K are respectively the damping, mass and stiffness matrices. For a given damping ratio η in the two modes, $\bar{\alpha}$ and $\bar{\beta}$ are given by:

$$\bar{\alpha} = \frac{4\pi}{T_1+T_2}\eta \;;\; \bar{\beta} = \frac{T_1 T_2}{\pi(T_1+T_2)}\eta \qquad (6)$$

where T_1 and T_2 are the two natural vibration periods of the system.

Due to the irregular nature of $\ddot{U}_g(t)$, the maxima of the response values of interest can only be obtained by means of step-by-step (i.e. time history) solution of equation 4. Since y_{max} or the maximum displacement $\bar{y}_{max}$ at CR and θ_{max} are unlikely to occur simultaneously, the maximum displacement of member i located at a distance $\bar{a}_i$ from CR, cannot be obtained from the forementioned maxima by simple superposition, i.e.:

$$y_{i,max} \neq \bar{y}_{max} + \bar{a}_i\, \theta_{max} \tag{7}$$

The correct procedure, of course, is to evaluate $y_{i,max}$ from its time history, namely:

$$y_{i,max} = (\bar{y}(t) + \bar{a}_i\, \theta(t))_{max} \tag{8}$$

The dynamic eccentricity e_d, in terms of the time history analysis, can be defined either as:

$$e_d = \frac{M_{max}}{V_{max}} = \frac{K_{\theta 0}\, \theta_{max}}{K_y\, \bar{y}_{max}} \tag{9}$$

or as:

$$e_d = \frac{M_{max}}{V_{o,max}} = \frac{K_{\theta 0}\, \theta_{max}}{K_y\, y_{o,max}} \tag{10}$$

in which V_{max} and M_{max} are respectively the maximum base shear and torque, and $V_{o,max}$ and y_o are respectively the maximum base shear and displacement in a single DoF system with a circular frequency equal to ω_y and the same damping ratio. The second definition of e_d is perhaps more relevant to earthquake codes, since the base shear to be taken is usually that of the associated symmetrical case, i.e. $V_{o,max}$.

When the static approach is used, the displacement at location i is computed from the following expression:

$$y_i = \frac{V_o}{K_y} + \frac{e_d\, V_o}{K_{\theta 0}}\, \bar{a}_i \tag{11}$$

in which V_o = the base shear. It is thus seen that the static code procedure is equivalent to using equation 7, rather than the more correct equation 8.

In the following section the implications of this incorrect evaluation of the maximum response in code oriented formulations are examined by comparing the static results computed by means of equation (11) with those obtained from time history analyses of several earthquake records.

NUMERICAL RESULTS

Time history analyses were performed using the computer program DRAIN 2D with an integration time step Δt = 0.01 seconds, for a number of 2-DoF systems of the type shown in Fig. 1. These had lateral natural periods T_o = 0.25, 0.50, 0.75; 1.00, 1.25, 1.50 and 2.00 seconds, and eccentricity ratios $e^* = e/\rho$ = 0.1, 0.3, 0.5, 0.7, 0.9, and 1.1. These systems were excited by five earthquake acceleration time histories:

El Centro: 1934 NS, 1940 NS, 1940 EW; Olympia 1949 N80E and Taft 1952 N69W. Five percent damping was assumed for the two coupled modes. Torsional to lateral frequency ratio was taken as $\Omega_o = 1.0$ for the full set of parameters. More limited studies were carried out for $\Omega_o^2 = 2.0$ and 0.5, characterizing respectively systems with high and low torsional rigidities. These had lateral natural periods $T_o = 0.25$, 1.00 and 2.00 seconds, and the same eccentricities as for $\Omega_o = 1.0$. The earthquake time histories used were: El Centro: 1940 NS, 1940 EW, and Olympia 1949 N80E.

It will be observed that the choice of Ω_o^2 rather than Ω_M^2 or Ω_R^2 ($= \Omega_o^2/[1+e^{*2}]$), commonly used by investigators, as the frequency ratio parameter is based on the observation that this frequency ratio is independent of eccentricity, i.e. by assuming it to be constant while varying the eccentricity, it is possible to isolate the effect of eccentricity from all other properties of the system. With the other definitions, however, a constant frequency ratio implies that variations in eccentricity are accompanied by changes in stiffness or mass related parameters.

Maximum lateral displacements at several stations along the x-axis of the floor deck were computed, as well as the maximum rotations θ, about the vertical axis. These responses were "normalized" by dividing the displacements $y_{i,max}$ through the spectral displacements $y_{o,max}$ for the respective translational periods of the corresponding earthquake time history and damping ratio, i.e. by the lateral response of associated systems having e=0, but otherwise identical.

Because of space limitations, only partial results are reported, and these refer mainly to members located at $a = \pm 1.5\rho$ from CM. These stations represent the two opposite edges of a rectangular building with an aspect ratio $d/b \simeq 0.6$ (Fig. 1). Limited results for members located at $a = + 1.0\rho$ are also presented. For a given station and frequency ratio Ω_o, the normalized maximum lateral displacements as computed by five different procedures are compared for the range of eccentricity ratios e^*. Note that for the upper limit $e^* = 1.1$, $e = 0.37b$ when $b = 3\rho$, which is sufficiently large. In Figs. 2,3 and 4, the time history results are presented by their average value and by the average + 1.0σ (σ = standard deviation). These are compared with several static analyses. The curve denoted "STATIC" is for $e_d = e$, and the one denoted "NBCC 1980" shows the results based on the provisions of the National Building Code of Canada 1980 (3) with $\beta=0$ (eq. 1), i.e. the effects considered are only those computed by means of equation 4. Comparison is also made with the formula for dynamic eccentricity recently proposed by Dempsey and Tso (5), which is denoted in Fig. 2 as "D&T". For $b=3\rho$ their formula takes the form:

$$
\begin{aligned}
e_d^* &= 3e^* \; ; \; e^* < 0.12 \\
e_d^* &= 0.36 + 0.85\,(e^* - 0.12) \; ; \; e^* \geq 0.12
\end{aligned}
\qquad (12)
$$

The different effects of increasing eccentricity and frequency ratio on the responses at the stations considered here are immediately apparent. The behaviour at a = + 1.5ρ (Fig. 3) is perhaps the most interesting. Whereas the displacements at -1.5ρ at all eccentricities are higher than for e=0, this is not the case at +1.5ρ for Ω_o^2 = 0.5 and 1.0, where the response is higher for small e* and lower for higher e*. However, for torsionally stiffer systems (Ω_o^2 = 2.0), the picture is quite different.

When the "AVERAGE + 1.0σ" is taken as the standard for comparison with the static methods, it is only to be expected that the "STATIC" (i.e. e_d = e) approach would underestimate the response. This usually is also true for other stations along the deck (not shown). Considering the "NBCC 1980" results, it is seen that the agreement with time history analysis is perhaps tolerable at the flexible edge of the building (a = - 1.5ρ), but is quite poor at the stiff edge (a = + 1.5ρ), for Ω_o^2 = 0.5 and 1.0. For a = - 1.5ρ the estimates given by the NBCC improve with falling frequency ratio, provided the requirement for doubling the adverse effects of eccentricity when e_d > 0.25b is ignored. However, for small eccentricities the NBCC still underestimates the response when Ω_o^2 = 1.0 (Fig. 2b), as is well known. The results based on equation 12, i.e. "D&T" in Fig. 2, follow the same trend as those of the NBCC, with the important exception that the response for smaller eccentricities is better estimated.

The inability of the NBCC to predict the response at +1.5ρ for Ω_o^2 = 0.5 and 1.0 is evident (Figs. 3a and 3b). Its estimates improve, however, when Ω_o^2 = 2.0 (Fig. 3c), and perhaps to a lesser extent for a = + 1.0ρ (Fig. 4). Note that Dempsey and Tso (5) did not propose a new formula to replace the NBCC expression, but suggested to resort to the traditional practice of ignoring the negative torsional contribution. However, even this procedure is very unconservative for small e* in torsionally flexible systems (Ω_o^2 = 0.5), and may become somewhat conservative in torsionally rigid ones. Yet, Fig. 4 suggests that, for members closer to CM than 1.5ρ, the latter observation does not apply for the larger eccentricities.

The effect of changes in the lateral vibration period T_o on the response was also examined. Although some differences between the maximum displacements of systems with natural periods T_o = 0.25, 1.00 and 2.00 seconds were found, no systematic variation of response with period could be discerned. Note, however, that somewhat better correlation with static results was observed for the stiffest systems (T_o = 0.25).

CONCLUDING REMARKS

Traditionally, the main effort of code oriented research on asymmetric buildings focused on the dynamic amplification of torsion in structures having close lateral and torsional frequencies ($\Omega_o \simeq 1.0$). However, as shown in recent studies (e.g. refs. 4,5) the derived dynamic eccentricities led to acceptable predictions of earthquake response by means of statics only for the flexible edge of the building (a = 1.5ρ in Fig. 2b). The results of the present study suggest that the frequency ratio Ω_o is an important parameter, so that predictions based on systems with

$\Omega_o = 1.0$ are likely to give poor estimates for the response of other systems. In particular, large deviations from static estimates are likely to occur at the stiff edge ($a = + 1.5\rho$) of torsionally flexible systems ($\Omega_o = 0.71$), and at the flexible edge ($a = - 1.5\rho$) of torsionally rigid systems ($\Omega_o = 1.41$).

These results also suggest that it may be difficult to cover by means of a single, yet simple, static expression the effects of eccentricity and frequency ratio on the response of members located at any given station along the floor deck. In view of the limitations inherent in the static approach, it appears that simple spectral analysis of two degree-of-freedom models representing the asymmetry of the system described in ref. 4 may become more attractive. Yet, if static procedures are to remain in use, the concept of dynamic eccentricity will have to be refined so that eccentricity, frequency ratio, and member location will be considered. In this respect, the formula proposed in ref. 5 is a step in the right direction. However, if it is meant to replace the design eccentricity expression in building codes for members located on the flexible side of floor it requires suitable factoring to cover systems having high torsional rigidity. Providing a more realistic expression for the stiff edge is more problematic, since the traditional rule of ignoring the negative effects of torsion is not adequate for most cases. Nevertheless, it is believed that, as an interim measure, this rule should be adopted by the codes, together with a provision requiring dynamic analysis for torsionally flexible systems.

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LIST OF FIGURES

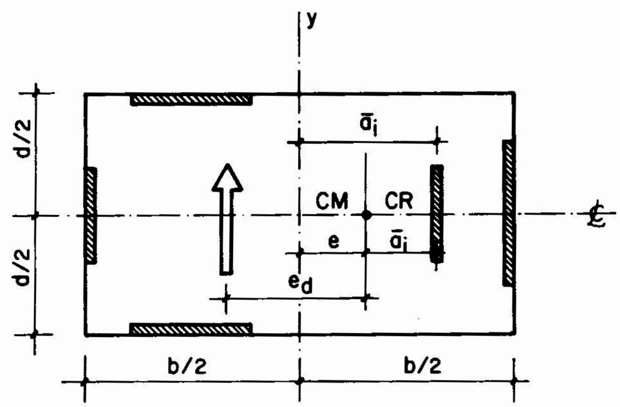

Fig. 1: Plan of Single Storey Structural Model.

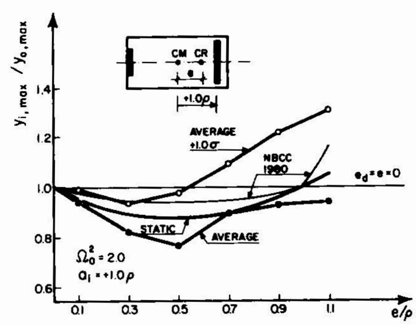

Fig. 4: Comparison of Results: Lateral Displacement vs. Eccentricity Ratio at a = + 1.0ρ.

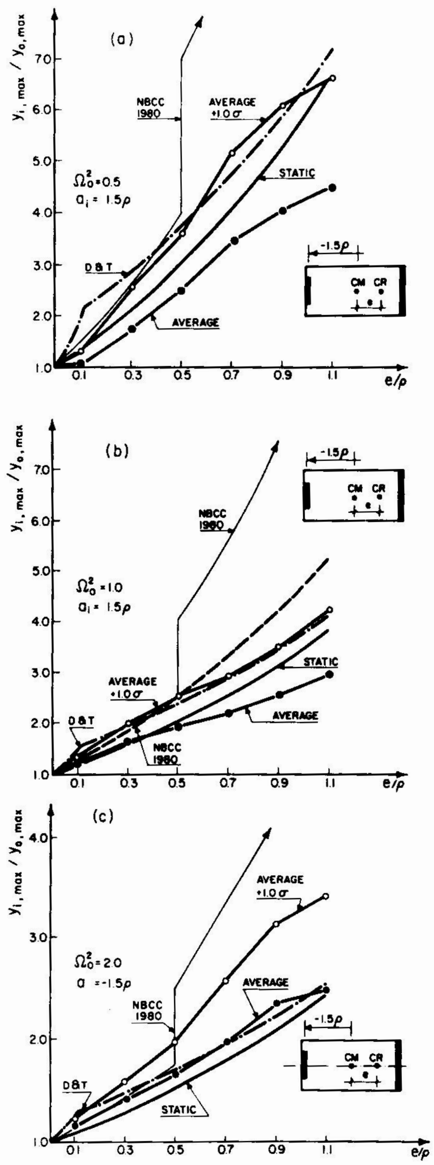

Fig. 2: Comparison of Results: Lateral Displacement vs. Eccentricity Ratio at a = - 1.5ρ.

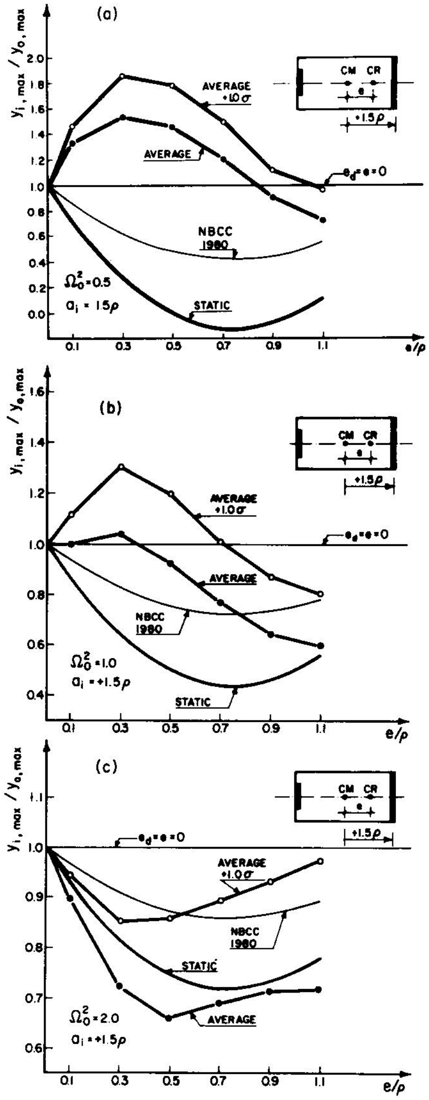

Fig. 3: Comparison of Results: Lateral Displacement vs. Eccentricity Ratio at a = + 1.5ρ.